Simple model for the input impedance of coax-fed rectangular microstrip patch antenna for CAD

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Abstract: We present a simple model for the input impedance of a rectangular microstrip patch antenna. This model is well suited for computer aided design (CAD). It is based on classical methods: (a) the cavity model determining the frequency and the input resistance at resonance, (b) the dynamic permittivity of a rectangular microstrip patch antenna (to take into account the influence of the fringing field at the edges of the rectangular patch antenna) and (c) the resonant parallel *RLC* circuit with an inductive reactance. This model is valid for electrically thick substrates. The theoretical results are in good agreement with experimental data.

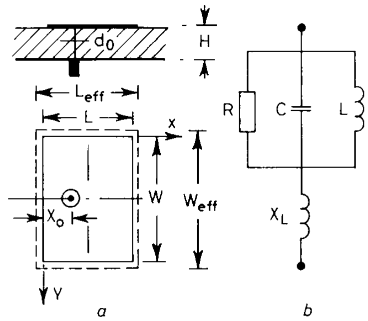

Fig. 1 *(a) Geometry of a rectangular microstrip patch antenna. (b) Equivalent resonant parallel RLC circuit*

1 Introduction

Rigorous methods developed by a number of authors [1–3] enable computation of the input impedance and the radiation pattern of rectangular microstrip patch antenna with good precision.

However, these methods involve complex analysis of the physical phenomena, which often take considerable computation time and do not easily yield the equivalent circuit. Sometimes, less accurate results suffice, and can be obtained much faster with the help of simpler methods such as the cavity model [4–7], which adequately describe the resonant frequency, input impedance, bandwidth and radiation pattern by a simple design equation. An analytical expression is given here for the imput impedance of a rectangular microstrip patch antenna excited by a coaxial probe (Fig. 1*a*) using the cavity model and the equivalent resonant circuits. It shows explicitly the dependence of the input impedance on the characteristic parameters of a patch antenna, and is valid for electrically thin and thick substrates.

2 Analysis

The cavity model is a bidimensional model that can be applied to patches whose geometries are specified simply by curvilinear orthogonal co-ordinate systems (rectangular, circular, etc.). It is possible to consider either the dominant mode or the complete spectrum of modes.

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The rectangular microstrip patch antenna can be considered in the fundamental mode, modelled by a simple resonant parallel *RLC* circuit (Fig. 1*b*). To take the coax-feed probe into account, it is necessary to modify the input impedance by an inductive reactance term [8]

$$X_L = \frac{377 f H}{c_o} \operatorname{Ln}\left(\frac{c_o}{\pi f d_o \sqrt{\varepsilon_r}}\right) \tag{1}$$

where c_o is the velocity of light in vacuum and d_o is the diameter of the probe. The input impedance is then obtained as

$$Z(f) = \frac{R}{1 + Q_T^2\left[\frac{f}{f_R} - \frac{f_R}{f}\right]^2} + j\left[X_L - \frac{RQ_T\left[\frac{f}{f_R} - \frac{f_R}{f}\right]}{1 + Q_T^2\left[\frac{f}{f_R} - \frac{f_R}{f}\right]^2}\right] \tag{2}$$

R is the resonant resistance of the resonant parallel *RLC* circuit [4] given in eqn. 3, in which we replace the effective permittivity ε_{eff} by the dynamic permittivity ε_{dyn} to take into consideration the influence of the fringing field at the edges of the patch, i.e.

$$R = \frac{Q_T H}{\pi f_R \varepsilon_{dyn} \varepsilon_o L W} \cos^2\left(\frac{\pi X_o}{L}\right) \tag{3}$$

The dynamic permittivity ε_{dyn} is a function of the dimensions (W, L, H) of the relative permittivity ε_r and the different modes field distribution [9]:

$$\varepsilon_{dyn} = C_{dyn}(\varepsilon)/C_{dyn}(\varepsilon_o) \tag{4}$$

where $C_{dyn}(\varepsilon)$ represents the total dynamic capacitance of the patch in the presence of a dielectric of relative permittivity $\varepsilon_r = \varepsilon/\varepsilon_o$ and $C_{dyn}(\varepsilon_o)$ represents the total dynamic capacitance of the patch in the presence of air. The total dynamic capacitance $C_{dyn}(\varepsilon)$ in the presence of a dielectric of relative permittivity ε_r can be written as

$$C_{dyn}(\varepsilon) = C_{o,\,dyn}(\varepsilon) + 2C_{e1,\,dyn}(\varepsilon) + 2C_{e2,\,dyn}(\varepsilon) \quad (5)$$

where $C_{o,\,dyn}(\varepsilon)$ is the dynamic main field of the patch capacitance without considering the fringing field. This can be calculated by

$$C_{o,\,dyn}(\varepsilon) = \frac{\varepsilon_o \varepsilon_r WL}{H\gamma_n\gamma_m} = \frac{C_{o,\,stat}(\varepsilon)}{\gamma_n \gamma_m} \quad (6)$$

where $C_{o,\,stat}(\varepsilon)$ represents the static main capacitance of the patch without considering the fringing field and γ_n and γ_m are in the form:

$$\gamma_i = \begin{cases} 1 \text{ for } i = 0 \\ 2 \text{ for } i \neq 0 \end{cases} \quad (7)$$

Then, a dynamic edge capacitance for each side of the patch taking into account the influence of the fringing field is calculated. Assuming that the edge field of the resonator has an X- and Y-dependent field distribution, the dynamic fringing capacitances are then given in the general form by

$$C_{e1,\,dyn}(\varepsilon) = \frac{1}{L}\int_o^L C_{e1,\,stat}(\varepsilon)\cos^2\left(\frac{n\pi x}{L}\right)dx = \frac{1}{\gamma_n}C_{e1,\,stat}(\varepsilon) \quad (8a)$$

and

$$C_{e2,\,dyn}(\varepsilon) = \frac{1}{W}\int_o^W C_{e2,\,stat}(\varepsilon)\cos^2\left(\frac{m\pi y}{W}\right)dy = \frac{1}{\gamma_m}C_{e2,\,stat}(\varepsilon) \quad (8b)$$

where $C_{e1,\,stat}(\varepsilon)$ represents the edge capacitance on one side of a patch length L and $C_{e2,\,stat}(\varepsilon)$ represents the edge capacitance on one side of a patch width W. Then $C_{e1,\,stat}(\varepsilon)$ is given by

$$C_{e1,\,stat}(\varepsilon) = \frac{1}{2}\left[\frac{Z(W, H, \varepsilon_r = 1)}{c_o Z^2(W, H, \varepsilon_r)} - \frac{\varepsilon_o \varepsilon_r W}{H}\right]L \quad (9)$$

where $Z(W, H, \varepsilon_r)$ is the characteristic impedance of the microstrip line [10]. In addition, the effect of the strip thickness can be neglected when $t = 0$; $Z(W, H, \varepsilon_r)$ is thus given by

$$Z(W, H, \varepsilon_r) = \frac{377}{\sqrt{\varepsilon_{eff}(W)}}\left[\frac{W}{H} + 1.393 + 0.667\,\mathrm{Ln}\left(\frac{W}{H} + 1.444\right)\right]^{-1} \quad \text{when } \frac{W}{H} \geqslant 1 \quad (10)$$

To evaluate ε_{eff} the simple equation given by Schneider was used [11]:

$$\varepsilon_{eff}(W) = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + \frac{10}{W/H}\right)^{-1/2} \quad (11)$$

Similarly $C_{e2,\,stat}(\varepsilon)$ is given by

$$C_{e2,\,stat}(\varepsilon) = \frac{1}{2}\left[\frac{Z(L, H, \varepsilon_r = 1)}{c_o Z^2(L, H, \varepsilon_r)} - \frac{\varepsilon_o \varepsilon_r L}{H}\right]W \quad (12)$$

To obtain $C_{dyn}(\varepsilon_0)$, ε can be replaced by ε_o in all of the above equations.

Q_T is the quality factor associated with system losses, which include radiation from the walls (Q_R), losses in the dielectric (Q_D) and losses in the conductor (Q_C). Q_T may be calculated by

$$Q_T = \left[\frac{1}{Q_R} + \frac{1}{Q_C} + \frac{1}{Q_D}\right]^{-1} \quad (13)$$

where Q_R is given by eqn. 14 [12] in which we replace the relative permittivity by the dynamic permittivity to take into account the influence of the fringing field at the edges of the patch:

$$Q_R = \frac{c_o\sqrt{\varepsilon_{dyn}}}{4f_R H} \quad (14)$$

The dielectric loss, Q_D, is given by

$$Q_D = \frac{1}{Tg\delta} \quad (15)$$

where $Tg\delta$ is the dielectric loss tangent to the substrate.

The conductor losses can be calulated from the equations given by James [13]:

$$Q_C = \frac{0.786\sqrt{f_R}\,Zao(W)H}{P_a} \text{ for copper; } f_R \text{ in GHz} \quad (16)$$

where $Zao(W)$ is the impedance of an air filled microstrip line of width W and thickness H. It is evaluated from $Za(W)$ (given below) by setting $\varepsilon_r = 1$. $Za(W)$ is the impedance of a dielectric filled line [13]:

$$Za(W) = \frac{60\pi}{\sqrt{\varepsilon_r}}\left\{\frac{W}{2H} + 0.441 + 0.082\left[\frac{\varepsilon_r - 1}{\varepsilon_r^2}\right] + \frac{(\varepsilon_r + 1)}{2\pi\varepsilon_r}\left[1.451 + \mathrm{Ln}\left(\frac{W}{2H} + 0.94\right)\right]\right\}^{-1} \quad \text{when } W/H > 1 \quad (17)$$

and

$$P_a(W) = \frac{2\pi\left[\frac{W}{H} + \frac{W/(\pi H)}{W/(2H) + 0.94}\right]\left[1 + \frac{H}{W}\right]}{\left\{\frac{W}{H} + \frac{2}{\pi}\mathrm{Ln}\left[2\pi e\left(\frac{W}{2H} + 0.94\right)\right]\right\}^2} \quad \text{when } W/H \geqslant 2 \quad (18)$$

f_R is the resonant frequency of a rectangular microstrip patch antenna with a larger width W and a longer length L, both comparable to $\lambda/2$, where λ is the wavelength in the substrate [14]. The resonant frequency corresponds to the frequency for which the real part of the input impedance is maximum, the additive reactance term X_L does not modify the value of the resonant frequency:

$$f_R = f_{mn} = \frac{c_o}{2\sqrt{\varepsilon_{dyn}}}\sqrt{\left[\left(\frac{m}{W_{eff}}\right)^2 + \left(\frac{n}{L_{eff}}\right)^2\right]} \quad (19)$$

where W_{eff} and L_{eff} [14] are the effective width and length, respectively, taking into account the influence of the fringing field at the corners and the dielectric inhomogeneity of the rectangular microstrip patch antenna.

We can calculate L_{eff} from the following relation:

$$L_{eff} = L + \left(\frac{W_{eq} - W}{2}\right) \frac{\varepsilon_{eff}(W) + 0.3}{\varepsilon_{eff}(W) - 0.258} \quad (20)$$

where W_{eq} is the equivalent width [5] calculated from the planar waveguide model:

$$W_{eq} = \frac{120\pi H}{Za(W)\sqrt{\varepsilon_{eff}(W)}} \quad (21)$$

Similarly, we can calculate W_{eff} from eqns. 20 and 21 by replacing L_{eff}, L, W_{eq} and W with W_{eff}, W, L_{eq} and L, respectively.

3 Results

In this Section computations concerning the fundamental mode ($m = 0$, $n = 1$) are presented and compared with measurements and previous computations. In Table 1 we present computed and measured values of the resonant frequency for thin substrates. We observe that our results are equal to or better than those predicted by Sengupta [15], and are in good agreement with experiment. In Table 2 we compare our computed values of the resonant frequency for thick substrates in various antenna geometries [18] with theoretical and experimental results obtained by other scientists. James's values are smaller than experimental values whereas those of Hammerstad are greater (except when $W = 1.7$ cm, $L = 1.1$ cm and $H = 0.1524$ cm). Our model predicts resonant frequencies closer to experimental values for most cases other than the first two geometries. An overall accuracy of approximately 3% was found for the thickness H, such that $H/\lambda < 0.23$. Also, an upper limit for ε_r is 10.

Table 1: Computed and measured values of resonant frequency for thin substrates, mode ($m = 0, n = 1$)

W (cm)	L (cm)	ε_{eff}	f_o (GHz)	Measured (GHz) [16]	Computed (GHz) [15]	Model (GHz)
4.100	4.140	2.390	2.343	2.228	2.248	2.245
6.858	4.140	2.428	2.325	2.200	2.228	2.221
10.800	4.140	2.452	2.314	2.181	2.216	2.204
11.049	6.909	2.453	1.386	1.344	1.347	1.347

$H = 0.1524$ cm; $\varepsilon_r = 2.5$; W and L variable; $f_o = c_o/2L\sqrt{\varepsilon_{eff}}$

Table 2: Computed and measured values of resonant frequency for thick substrates, mode ($m = 0, n = 1$)

W (cm)	L (cm)	H (cm)	Measured (GHz)	James (GHz)	Hammerstad GHz)	Model (GHz)
5.70	3.80	0.3175	2.31	2.30	2.38	2.38
4.55	3.05	0.3175	2.89	2.79	2.90	2.91
2.95	1.95	0.3175	4.24	4.11	4.34	4.29
1.95	1.30	0.3175	5.84	5.70	6.12	5.96
1.70	1.10	0.3175	6.80	6.47	7.01	6.76
1.40	0.90	0.3175	7.70	7.46	8.19	7.82
1.20	0.80	0.3175	8.27	8.13	9.01	8.50
1.05	0.70	0.3175	9.14	8.89	9.97	9.30
0.90	0.60	0.3175	10.25	9.82	11.18	10.27
1.70	1.10	0.1524	7.87	7.46	7.84	7.79
1.70	1.10	0.3175	6.80	6.47	7.01	6.76
1.70	1.10	0.9525	4.73	4.32	5.27	4.52

$\varepsilon_r = 2.33$

Fig. 2 shows the input impedance for a patch operating at about 0.66 GHz. The results computed by Pozar [19] and the measurements of Lo [20] are reported. Fig. 3 shows the input impedance for a patch operating at about 2.22 GHz. Our computations are compared with the computed results and measurements of Carver [7, 17]. Fig. 4 presents the input impedance for a patch operating at about 4.44 GHz. We compare our computations (where the feed is located at the midpoint of the longer

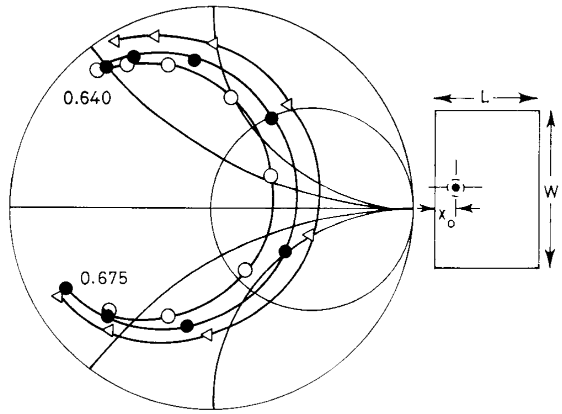

Fig. 2 *Input impedance of coax-fed microstrip patch antenna*

$\varepsilon_r = 2.59$; $Tg\delta = 0.003$; $H = 0.1588$ cm; $d_o = 0.127$ cm; $Zo = 50\ \Omega$; ● measured [20]; △ calculated [19]; ○ our model; mode ($m = 0$, $n = 1$); $L = 13.97$ cm; $W = 20.45$ cm; $Xo = 0.635$ cm

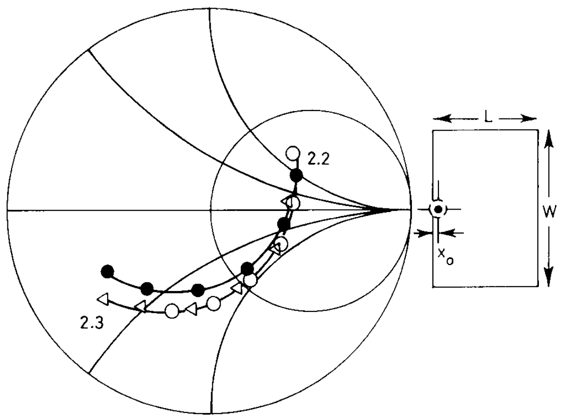

Fig. 3 *Input impedance of coax-fed microstrip patch antenna*

$\varepsilon_r = 2.50$; $Tg\delta = 0.002$; $H = 0.1524$ cm; $d_o = 0.127$ cm; $Zo = 50\ \Omega$; ● measured [7]; △ calculated [7]; ○ our model; mode ($m = 0$, $n = 1$); $L = 4.140$ cm; $W = 6.858$ cm; $Xo = 0.0$

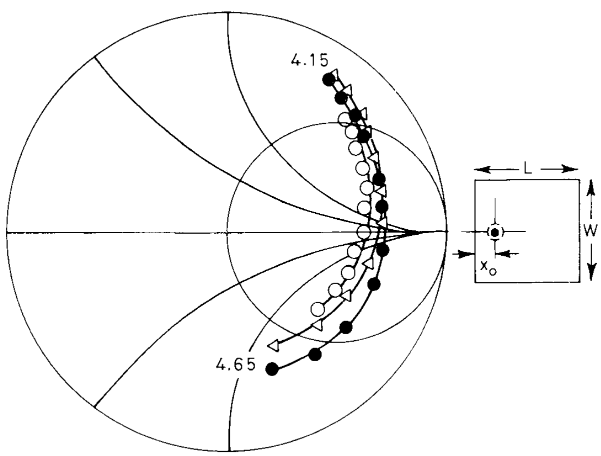

Fig. 4 *Input impedance of coax-fed microstrip patch antenna*

$\varepsilon_r = 2.55$; $Tg\delta = 0.002$; $H = 0.159$ cm; $d_o = 0.127$ cm; $Zo = 50\ \Omega$; ● measured [8]; △ calculated [8]; ○ our model; mode ($m = 0$, $n = 1$); $L = 2.01$ cm; $W = 2.01$ cm; $Xo = 0.13$ cm side $Yo = W/2$ cm, $Xo = 0.13$ cm) with the computed results and the measurements of Bailey [8] (where the feed is located at $Yo = 1$ cm, $Xo = 0.13$ cm). It is emphasized that Figs. 2–4 are for thin substrates ($H/\lambda \sim 0.02$). However, the thicker substrates ($H/\lambda \sim 0.08$) we present in Fig. 5 the input impedance of a patch operating at

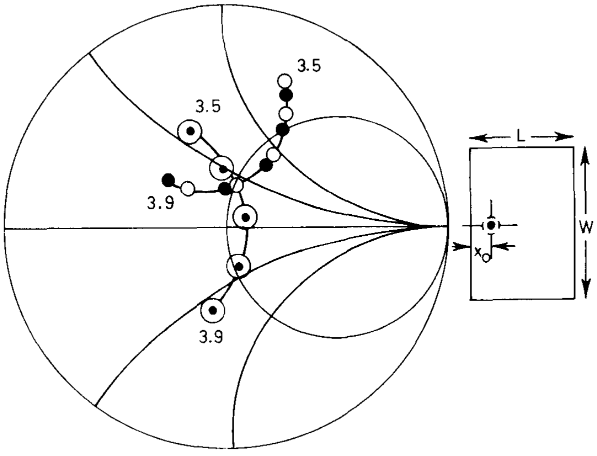

Fig. 5 *Input impedance of coax-fed microstrip patch antenna*

$\varepsilon_r = 4.53$; $Tg\delta = 0.025$; $H = 0.300$ cm; $d_o = 0.065$ cm; $Zo = 50\ \Omega$; ● measured; ○ our model with X_L; ⊙ our model without X_L; mode ($m = 0$, $n = 1$); $L = 1.74$ cm; $W = 2.31$ cm; $Xo = 0.55$ cm

about 3.72 GHz where we show the influence of the inductive reactance term X_L. When the term X_L is considered, our calculated values are in good agreement with our measured values. If X_L is neglected, on the other hand, the circular input impedance locus is centred on the real axis of the Smith chart. Thus it is necessary to consider the contribution of the coaxial probe.

4 Conclusion

We have developed a simple model yielding the input impedance of a probe-fed rectangular microstrip patch antenna, which gives results in accordance with experiment. This model can be used successfully in computer aided design (CAD) of rectangular microstrip antenna arrays.

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